Paolo Mancosu, The adventure of reason: interplay between philosophy of mathematics and mathematical logic, 1900–1940

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Paolo Mancosu, *The Adventure of Reason. Interplay between Philosophy of Mathematics and Mathematical Logic, 1900-1940*, Oxford University Press, 2010, 640 pp, £60, ISBN 978-0-19-954653-4

Throughout his career Professor Paolo Mancosu has held a keen interest in the philosophy and history of logic. In *The Adventure of Reason* 17 of Mancosu's papers exploring the relationship between philosophy and the development of mathematical logic have been collected and published with revised bibliography and introductory comments by the author.

The book is divided into five parts. The first part consists of a historical overview over the development of relevant parts of mathematical logic during the first four decades of the 20th century. It covers the various attempts at axiomatizing arithmetic and set theory (Peano, Russell, Hilbert, Zermelo), the development of the Löwenheim-Skolem theorem, Gödel's incompleteness theorems, the discussion over intuitionistic logic and the development of semantics. The essay is not intended to be a complete history of the development of mathematical logic during the period in question. Rather, its aim is to add to the existing literature by filling in details and focusing on themes, often overlooked in usual textbook treatments of the subject. Thus for instance, Gödel is given a rather sparse treatment, while the development of the somewhat related Löwenheim-Skolem theorem is described in full detail. The essay can be read (and enjoyed) in its own right, but more importantly it serves the function of enabling the reader to set the themes discussed in the subsequent chapters in their right historical context.

The second part of the book is dedicated to foundational issues. It consists of seven papers focusing mainly on the development of Hilbert's program and the changes in Hilbert's conception of finitistic arithmetic, on the debate over the constructivity of proofs that took place during the 1930s, and on the reception of Gödel's incompleteness theorems.

In the third part of the book the almost forgotten discussion over the possibility of giving a phenomological account of the exact sciences is brought to light. This is done by recounting and analysing the correspondences that took place between Oskar Becker, Hermann Weyl and Dietrich Mahnke during the 1920's and early 1930's. In these letters, the intellectually rich world of especially Edmund Husserl's phenomenology is brought into contact with the discussions taking place in the philosophy of mathematics and physics in the period.

The fourth part of the book treats the development of nominalism during the first half of the 20th century. The section takes departure in the discussions that took place between Rudolf Carnap, Alfred Tarski and Wilard van Orman Quine while they were all at Harvard during the academic year 1940-41. From this starting point, the development of especially Tarski and Quine's position on the matter during the following years is traced.

The fifth and final part of the book is dedicated to Takski. His discussion with the Vienna Circle on semantics is recounted, and his precise conception of logical consequence is analysed. This final analysis includes two until now unpublished papers; the first is an unpublished lecture by Tarski on logical consequences and the second is Mancosu's detailed commentary to and analysis of this lecture.

The intended audience of *The Adventure of Reason* is those, who wants to know more. Although the historical overview goes some way of serving as an introduction, the reader will only profit fully from the book, if she has at least some knowledge of the topic in advance. Readers familiar with the philosophy of mathematics and the standard history of mathematical logic will, however, not be disappointed. The book adds important nuances and fils in interesting details to the basic story. This is achieved through the careful

analysis of central papers, but first and foremost by tapping into of the hidden wealth of knowledge - letters, diaries and unpublished manuscripts - that is stored away in various archives. The papers collected in the book reflect and is the result of careful archival work, where relevant, but long forgotten details of the intellectual lives of some of the main characters of the 20th century philosophy of mathematics and mathematical logic has been identified and uncovered. And that, no doubt, is the chief attraction of the book.

The Adventure of Reason can be read from cover to cover, but due to its nature as a collection of individual papers, it can also be read in selections according to the reader's interests. It is however highly recommended to read full parts of the book, and not singular papers. The five parts of the book are all very well-composed. The papers collected in each part all add details and shed further light on the theme in question. Thus, if the reader is interested in one of the papers of a part, she wil surely cheat herself, if she does not read the rest.

All in all, *The Adventure of Reason* is an example of excellent scholarly work. Those already familiar with the basic outline of the 20th century philosophy of mathematics will find in it a wealth of additional details and new nuances.

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